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BIRZEIT UNIVERSITY
MATHEMATICS DEPARTMENT

Second Exam **Stat 236** **Fall 2013**

Time : 1 hour

Name (بالعربية)..[illegible]..

Number..[illegible]................................

Instructor...Mohammad..Media...........

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Formulas:

- **Discrete Random Variable**

$$E(X) = \mu = \sum x f(x)$$

$$Var(X) = \sum (x-\mu)^2 f(x)$$

- **Binomial Probability Distribution**

$$P(X=x) = f(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$E(X) = np$, $\sigma(X) = \sqrt{np(1-p)}$

- **Poisson Probability**

$$f(x) = \frac{\mu^x e^{-\mu}}{x!}$$

- **Exponential Probability Distribution**

$$\boxed{f(x) = \frac{1}{\mu} e^{\frac{-x}{\mu}}}$$

- **Sampling Distribution of the mean**

$$E(\bar{x}) = \mu,\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \text{ or } \sigma_{\bar{x}} = \sqrt{\frac{N-n}{N-1}}\frac{\sigma}{\sqrt{n}}$$

Instructions:

1. Clearly write your name, student number, and instructor name in the space above.
2. There are 21 problems in 5 pages, each worth 2 points.
3. Please work each problem in the space provided. Show all calculations and display answers clearly. Unjustified answers will receive no credit
4. You can use your own calculator only.
5. Please be sure to turn your cell phone off.

(1 - 2) The demand for a product varies from month to month. Based on the past year's data, the following probability distribution shows ABC Company's monthly demand.

discreat

Unit Demand (x)	Probability f(x)	x F(x)	
0	0.1	0	
1000	0.1	100	
2000	0.3	600	
3000	0.4	1,200	
4000	0.1	400	
		2,300	

1. Determine the **expected number** of units demanded per month.

$E(x) = \sum x F(x) = \boxed{2,300}$

2. Each unit produced costs the company \$8, and is sold for \$10. How much will the company gain or lose in a month if they stock(يخزن) the expected number of units demanded, but sell 2000 units?

~~34,400~~

\$8 ⟹ \$10

$2000 * \$2 = \boxed{\$1000 \text{ gain}}$

3. A population of 1000 has a mean of 300 and a standard deviation of 20. A sample of 100 observations will be taken. Find the **standard error** of the sample mean.

Finite →

$\frac{100}{1000} > 0.05$

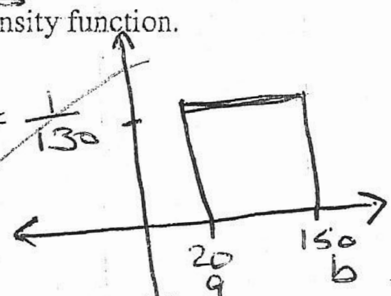
$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{20}{\sqrt{100}} = \boxed{2}$

(4 - 8) The length of time patients must wait to see a doctor in a local clinic is uniformly distributed between 20 minutes and 2.5 hours. 150 min.

4. Write and **graph** the probability density function.

$F(x) = \begin{cases} \frac{1}{130} \\ 0, \text{ eleswhere} \end{cases}$

$\frac{1}{b-a} = \frac{1}{130}$



5. What is the probability that a patient would have to wait between 45 minutes to 2 hours?

$x_2 = 2 \text{hour} \Rightarrow 160 \text{ min}$

$x_1 = 45$

$F(x) = \frac{x_2 - x_1}{b - a}$

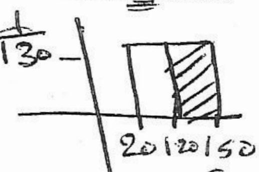
$= 120 - 45 = 75$

$(x > 120)$

6. Compute the probability that a patient would have to wait over 2 hours.

$F(x) = \dfrac{x_2 - x_1}{b - a}$

$= \dfrac{150 - 120}{150 - 20} = \dfrac{30}{130} = \boxed{.230}$



$x_1 = 120$

$x_2 = 150$

7. Determine the expected waiting time and its standard deviation.

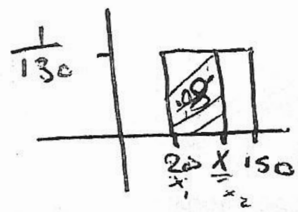
$E(x) = \dfrac{b+a}{2} = \dfrac{150+20}{2} = \dfrac{170}{2} = \boxed{85}$

$S.d(\sigma) = \dfrac{b-a}{\sqrt{12}} = \dfrac{150-20}{\sqrt{12}} = \dfrac{130}{\sqrt{12}} = \boxed{37.5}$

8. Find and interpret P_{80} for this distribution.

$.08 = \dfrac{x_2 - x_1}{150 - 20}$

$.08 = \dfrac{x_2 - 20}{130} \Rightarrow \boxed{x_2 = 30.4 \text{ min}}$



1/2

(9 – 11) A life insurance company has determined that each week an average of 3 claims(مطالبات) is filed in its Ramallah branch.

9. Find the probability that more than 2 claims filed in a week.

Poission

$\mu = 3$

$P(X>2) = 1 - (P(0) + P(1) + P(2))$

$= 1 - \left(\dfrac{3^0 e^{-3}}{0!} + \dfrac{3^1 e^{-3}}{1!} + \dfrac{3^2 e^{-3}}{2!}\right)$

$= 1 - (.0497 + .1493 + .2240)$

$= 1 - 0.423 = \boxed{0.577}$

10. Find the probability that exactly 10 claims filed in two weeks. $x = \boxed{10}$, $\mu = 3 \times 2 = \boxed{6}$

$P(X=10) = \dfrac{6^{10} e^{-6}}{10!} = \boxed{.0413}$

11. Suppose that the company accepts all claims , and the average cost of each claim is \$1000, what is the expected average cost in a year (52 weeks?

$3 \times 52 = 156 \times \$1000 = \boxed{\$156,000}$

expected average cost

115

1.048

(12 - 14) The student body of BZU consists of 60% female students.

discrete

12. A random sample of 8 students is selected. What is the probability that among the students in the sample at least 7 are female?

$X = B(8, .6)$ Binomial

$$P(X \geq 7) = P(7) + P(8)$$
$$= \binom{8}{7}(.6)^7(.4)^1 + \binom{8}{8}(.6)^8(.4)^0$$
$$= .0895 + 0.0167 = \boxed{0.1062}$$

13. A random sample of 10 students is selected. What is the probability that among the students in the sample at least 7 are male?

$X = B(10, .4)$

$$P(x \geq 7) = F(7) + F(8) + F(9) + F(10)$$
$$= \binom{10}{7}(.4)^7(.6)^3 + \binom{10}{8}(.4)^8(.6)^2 + \binom{10}{9}(.4)^9(.6)^1 + \binom{10}{10}(.4)^{10}(.6)^0$$
$$= .0424 + .0106 + 1.572 + .0001048 = \boxed{1.625}$$

14. A random sample of 120 students is selected. What is the approximated probability that among the students in the sample at least 80 are female?

$X = B(120, .6)$

$$\cancel{P(x \geq 80) = F(80) + F(90) + F(100) + F(110) + F(120)}$$
$$= \binom{120}{80}(.6)^{80}(.4)^{40} + \binom{120}{90}(.6)^{90}(.4)^{30} + \binom{120}{100}(.6)^{100}(.4)^{20} + \binom{120}{110}(.6)^{110}(.4)^{10} + \binom{120}{120}(.6)^{120}$$
$$= .0247 + 2.1145\times10^{-4} + 2.11656\times10^{-8} + 4.8078\times10^{-15} + 2.3886\times10^{-27}$$

$\boxed{\cancel{0.0249}}$

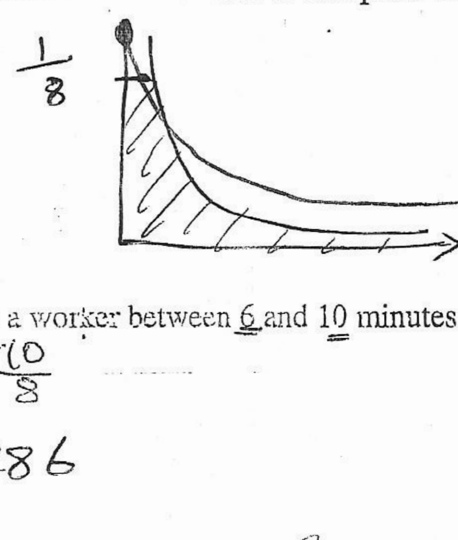
(15 - 16) The time it takes a worker on an assembly line to complete a task is exponentially distributed with a mean of 8 minutes.

15. What is the probability density function for the time it takes to complete the task? Graph the function.

$$F(x) = \frac{1}{\mu} e^{-x/\mu}$$

$$\boxed{F(x) = \frac{1}{8} e^{-\frac{x}{8}}}$$

$x > 0$



.75
1.25

16. What is the probability that it will take a worker between 6 and 10 minutes to complete the task?

$$P(6 < x < 10) = e^{-\frac{6}{8}} - e^{-\frac{10}{8}}$$
$$= .472 - .286$$
$$= \boxed{.186}$$

8.5

(17 – 19) ABC Corporation gives each of its employees an aptitude test. The scores on the test are normally distributed with a mean of 75 and a standard deviation of 15. A simple random sample of 25 is taken from a population of 500. N

17. What are the expected value, the standard deviation, and the shape of the sampling distribution of x?

$- E(x) = \mu = 75.$

$- \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{25}} = \frac{15}{5} = \boxed{3}$

$\frac{25}{500} = \boxed{0.05}$

- Shape of the Sampling distribution is Normally because its Large ⇒
N > 30
100 > 30

18. What is the probability that the average aptitude test in the sample will be between 70 and 82?

$P(70 < \bar{X} < 82) = P\left(\frac{70-75}{3} < Z < \frac{82-75}{3}\right)$

$= P(-1.6 < Z < 2.3)$

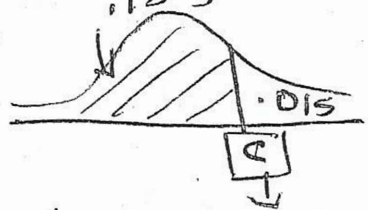
$= P(Z < 2.3) + P(Z < 1.6) - 1$

$= .9893 + .9452 - 1$

$= \boxed{.9345}$

19. Find a value, C, such that $P(\bar{x} \geq C) = .015$.

$P(x \geq C) = .015$



$Z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}}$

$2.17 = \frac{\bar{X} - 75}{3} \Rightarrow \boxed{\bar{X} = 81.5}$

$Z = 2.17$

20. The distribution of scores for a particular exam follows a normal distribution with mean of 70 and standard deviation of 6. If you got a score of 80 in this class, at what percentile is your score?

$P(x < 80) = P\left(Z < \frac{80-70}{6}\right) = P(Z < 1.66)$

$= \boxed{.9515}$

$\boxed{\text{with } P_{95}}$

21. (**Bonus**) The distribution of scores for a particular exam follows a normal distribution with mean of 70 and standard deviation of 4. Assuming that the top 10% of students get an A, the next 10% a B, and so on. If got an 80 in this class, what grate did you get?

28.8

75